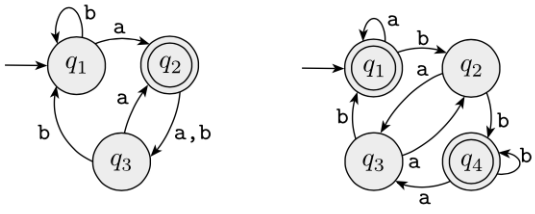
Exercise I, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

1 (*Exercise 1.1 in Sipser's book*) The following are the state diagrams of two DFAs, M_1 and M_2.



M_1 M_2

Answer the following questions about each of these machines.

1a What is the start state?

1b What is the set of accepting states?

1c What sequence of states does the machine go through on input aabb?

1d Does the machine accept the string aabb?

1e Does the machine accept the string ε?

2 (*Based on Exercise 1.4 in Sipser's book*) Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in class to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

2a $\{w \mid w$ has exactly two a's and at least two b's$\}$

2b $\{w \mid w$ has an even number of a's and each a is immediately followed by at least one b$\}$

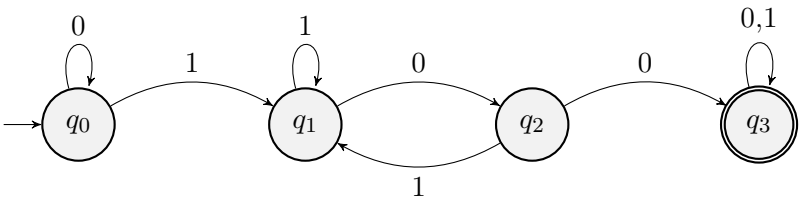
3 (*Based on Exercise 1.5 in Sipser's book*) Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

3a $\{w \mid w$ does not contain the substring ab$\}$

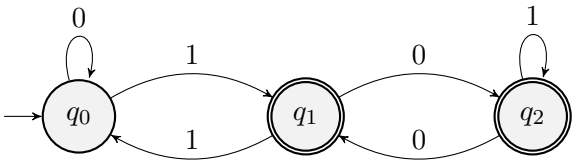
3b $\{w \mid w$ does not contain the substring baba$\}$

4 Suppose A_1, A_2, and A_3 are regular languages over the alphabet Σ. Prove that $(A_1 \cup A_2) \cap A_3$ is regular by giving a formal description of a finite automaton recognizing it.

5 For the automaton given below, describe the language it recognizes. Prove that your description is correct.



6 The following automaton is over the alphabet $\Sigma = \{0, 1\}$ and the set of states is $Q = \{q_0, q_1, q_2\}$. The start state is q_0 and the accepting states are $F = \{q_1, q_2\}$.



6a Write down the transition function δ for this automaton.

6b* Describe the language recognized by the automaton and prove the correctness of your claim.

Hint: *Try all inputs of a fixed length in some natural order and look for a pattern*